

# Package ‘bsamGP’

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**Type** Package

**Title** Bayesian Spectral Analysis Models using Gaussian Process Priors

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**Imports** MASS, ggplot2, gridExtra

**Description** Contains functions to perform Bayesian inference using a spectral analysis of Gaussian process priors. Gaussian processes are represented with a Fourier series based on cosine basis functions. Currently the package includes parametric linear models, partial linear additive models with/without shape restrictions, generalized linear additive models with/without shape restrictions, and density estimation model. To maximize computational efficiency, the actual Markov chain Monte Carlo sampling for each model is done using codes written in FORTRAN 90.

**License** GPL (>= 2)

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blq *Bayesian Quantile Regression*

---

### Description

This function fits a Bayesian quantile regression model.

### Usage

```
blq(y, w, p, mcmc = list(), prior = list(), marginal.likelihood = TRUE)
```

### Arguments

y	a vector of response values.
w	a vector or matrix giving covariates of dimension n times ndimw excluding intercept for a parametric component (w can be omitted)
p	quantile of interest (default=0.5).
mcmc	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): nblow (1000) giving the number of MCMC in transition period, nskip (1) giving the thinning interval, smcmc (1000) giving the number of MCMC for analysis.
prior	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): beta_m0 and beta_v0 giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, sigma2_m0 and sigma2_v0 giving the prior mean and variance of the inverse gamma prior for the scale parameter of response.
marginal.likelihood	a logical variable indicating whether the log marginal likelihood is calculated. The methods of Gelfand and Dey (1994) is used.

## Details

This generic function fits a Bayesian quantile regression model.

Let  $y_i$  and  $w_i$  be the response and the vector of parametric predictors, respectively. Further, let  $x_{i,k}$  be the covariate related to the response, linearly. The model is as follows.

$$y_i = w_i^T \beta + \epsilon_i, \quad i = 1, \dots, n,$$

where the error terms  $\{\epsilon_i\}$  are a random sample from an asymmetric Laplace distribution,  $ALD_p(0, \sigma^2)$ , which has the following probability density function:

$$ALD_p(\epsilon; \mu, \sigma^2) = \frac{p(1-p)}{\sigma^2} \exp\left(-\frac{(x-\mu)[p-I(x \leq \mu)]}{\sigma^2}\right),$$

where  $0 < p < 1$  is the skew parameter,  $\sigma^2 > 0$  is the scale parameter,  $-\infty < \mu < \infty$  is the location parameter, and  $I(\cdot)$  is the indication function.

The conjugate priors are assumed for  $\beta$  and  $\sigma$ :

$$\beta | \sigma \sim N(m_{0,\beta}, \sigma^2 V_{0,\beta}), \quad \sigma^2 \sim IG\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)$$

## Value

An object of class `blm` representing the Bayesian parametric linear model fit. Generic functions such as `print` and `fitted` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws`, the posterior samples of the fitted values are stored in the list `fit.draws`, and the MCMC samples for the log marginal likelihood are saved in the list `loglik.draws`. The output list also includes the following objects:

<code>post.est</code>	posterior estimates for all parameters in the model.
<code>lmargin</code>	log marginal likelihood using Gelfand-Dey method.
<code>rsquarey</code>	correlation between $y$ and $\hat{y}$ .
<code>call</code>	the matched call.
<code>mcmctime</code>	running time of Markov chain from <code>system.time()</code> .

## References

Gelfand, A. E. and Dey, K. K. (1994) Bayesian model choice: asymptotics and exact calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, **56**, 501-514.

Kozumi, H. and Kobayashi, G. (2011) Gibbs sampling methods for Bayesian quantile regression. *Journal of Statistical Computation and Simulation*, **81**(11), 1565-1578.

## See Also

[blr](#)

**Examples**

```
## Not run:
#####
# Simulated example #
#####

# Simulate data
set.seed(1)

n <- 100
w <- runif(n)
y <- 3 + 2*w + rald(n, scale = 0.8, p = 0.5)

# Fit median regression
fout <- blq(y = y, w = w, p = 0.5)

# Summary
print(fout)

# fitted values
fit <- fitted(fout)

# Plots
plot(fout)

## End(Not run)
```

blr

*Bayesian Linear Regression***Description**

This function fits a Bayesian linear regression model using scale invariant prior.

**Usage**

```
blr(y, w, mcmc = list(), prior = list(), marginal.likelihood = TRUE)
```

**Arguments**

y	a vector of response values.
w	a vector or matrix giving covariates of dimension n times ndimw
mcmc	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): nblow (1000) giving the number of MCMC in transition period, nskip (1) giving the thinning interval, smcmc (1000) giving the number of MCMC for analysis.
prior	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): beta_m0 and beta_v0 giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, sigma2_m0 and sigma2_v0 giving the prior mean and variance of the inverse gamma prior for the scale parameter of response.

`marginal.likelihood`

a logical variable indicating whether the log marginal likelihood is calculated.

## Details

This generic function fits a Bayesian linear regression model using scale invariant prior.

Let  $y_i$  and  $w_i$  be the response and the vector of parametric predictors, respectively. The model for regression function is as follows.

$$y_i = w_i^T \beta + \epsilon_i, \quad i = 1, \dots, n,$$

where the error terms  $\{\epsilon_i\}$  are a random sample from a normal distribution,  $N(0, \sigma^2)$ .

The conjugate priors are assumed for  $\beta$  and  $\sigma$ :

$$\beta | \sigma \sim N(m_{0,\beta}, \sigma^2 V_{0,\beta}), \quad \sigma^2 \sim IG\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)$$

## Value

An object of class `blm` representing the Bayesian spectral analysis model fit. Generic functions such as `print` and `fitted` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws` and the posterior samples of the fitted values are stored in the list `fit.draws`. The output list also includes the following objects:

<code>post.est</code>	posterior estimates for all parameters in the model.
<code>lmarg</code>	log marginal likelihood.
<code>rsquarey</code>	correlation between $y$ and $\hat{y}$ .
<code>call</code>	the matched call.
<code>mcmctime</code>	running time of Markov chain from <code>system.time()</code> .

## See Also

[blq](#)

## Examples

```
## Not run:
#####
# Simulated example #
#####

# Simulate data
set.seed(1)

n <- 100
w <- runif(n)
y <- 3 + 2*w + rnorm(n, sd = 0.8)

# Fit the model with default priors and mcmc parameters
fout <- blr(y = y, w = w)

# Summary
print(fout)
```

```

# Fitted values
fit <- fitted(fout)

# Plots
plot(fout)

## End(Not run)

```

---

bsad

*Bayesian Semiparametric Density Estimation*


---

### Description

This function fits a semiparametric model, which consists of parametric and nonparametric components, for estimating density using a logistic Gaussian process.

### Usage

```

bsad(x, xmin, xmax, nint, MaxNCos, mcmc = list(), prior = list(),
smoother = c('geometric', 'algebraic'),
parametric = c('none', 'normal', 'gamma', 'laplace'), marginal.likelihood = TRUE)

```

### Arguments

x	a vector giving the data from which the density estimate is to be computed.
xmin	minimum value of x.
xmax	maximum value of x.
nint	number of grid points for plots (need to be odd). The default is 201.
MaxNCos	maximum number of Fourier coefficients.
mcmc	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): <code>kappaloop</code> (5) giving the number of MCMC loops within each choice of kappa, <code>nblow</code> (10000) giving the number of MCMC in transition period, <code>nskip</code> (10) giving the thinning interval, <code>smcmc</code> (1000) giving the number of MCMC for analysis, and <code>ndisp</code> (1000) giving the number of saved draws to be displayed on screen (the function reports on the screen when every <code>ndisp</code> iterations have been carried out).
prior	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): <code>gmax</code> giving maximum value for gamma (default = 5), <code>PriorProbs</code> giving prior probability of parametric and semiparametric models, <code>beta_m0</code> and <code>beta_v0</code> giving the hyperparameters for prior distribution of the parametric coefficients, <code>r0</code> and <code>s0</code> giving the hyperparameters of $\sigma^2$ for the logits, <code>u0</code> and <code>v0</code> giving the hyperparameters of $\tau^2$ for Fourier coefficients, <code>PriorKappa</code> and <code>KappaGrid</code> giving prior on the number of cosine terms.
smoother	types of smoothing priors for Fourier coefficients. See Details.
parametric	specifying a distribution of the parametric part to be test.
marginal.likelihood	a logical variable indicating whether the log marginal likelihood is calculated.

## Details

This generic function fits a semiparametric model, which consists of parametric and nonparametric, for density estimation (Lenk, 2003):

$$f(x|\beta, Z) = \frac{\exp[h(x)^\top \beta + Z(x)]}{\int_{\mathcal{X}} \exp[h(y)^\top \beta + Z(y)] dG(y)}$$

where  $Z$  is a zero mean, second-order Gaussian process with bounded, continuous covariance function. i.e.,

$$E[Z(x), Z(y)] = \sigma(x, y), \quad \int_{\mathcal{X}} Z dG = 0 \quad (a.s.)$$

Using the Karhunen-Loeve Expansion,  $Z$  is represented as infinite series with random coefficients

$$Z(x) = \sum_{j=1}^{\infty} \theta_j \varphi_j(x),$$

where  $\{\varphi_j\}$  is the cosine basis,  $\varphi_j(x) = \sqrt{2} \cos[j\pi G(x)]$ .

For the random Fourier coefficients of the expansion, two smoother priors are assumed (optional),

$$\theta_j | \tau, \gamma \sim N(0, \tau^2 \exp[-j\gamma]), \quad j \geq 1 \quad (\text{geometric smoother})$$

$$\theta_j | \tau, \gamma \sim N(0, \tau^2 \exp[-\ln(j+1)\gamma]), \quad j \geq 1 \quad (\text{algebraic smoother})$$

The coefficient  $\beta$  have the popular normal prior,

$$\beta | m_{0,\beta}, V_{0,\beta} \sim N(m_{0,\beta}, V_{0,\beta})$$

To complete the model specification, independent hyper priors are assumed,

$$\tau^2 | r_0, s_0 \sim IGa(r_0/2, s_0/2)$$

$$\gamma | w_0 \sim Exp(w_0)$$

Note that the posterior algorithm is based on computing a discrete version of the likelihood over a fine mesh on  $\mathcal{X}$ .

## Value

An object of class `bsad` representing the Bayesian spectral analysis density estimation model fit. Generic functions such as `print`, `fitted` and `plot` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws`, the posterior samples of the fitted values are stored in the list `fit.draws`, and the MCMC samples for the log marginal likelihood are saved in the list `loglik.draws`. The output list also includes the following objects:

<code>post.est</code>	posterior estimates for all parameters in the model.
<code>lmarg</code>	log marginal likelihood.
<code>ProbProbs</code>	posterior probability of models.
<code>call</code>	the matched call.
<code>mcmctime</code>	running time of Markov chain from <code>system.time()</code> .

## References

Lenk, P. (2003) Bayesian semiparametric density estimation and model verification using a logistic Gaussian process. *Journal of Computational and Graphical Statistics*, **12**, 548-565.

## Examples

```
## Not run:
#####
# Old Faithful geyser data #
#####
data(faithful)
attach(faithful)

# mcmc parameters
mcmc <- list(nblow = 10000,
            smcmc = 1000,
            nskip = 10,
            ndisp = 1000,
            kappaloop = 5)

# fits BSAD model
fout <- bsad(x = eruptions, xmin = 0, xmax = 8, nint = 501, mcmc = mcmc,
            smoother = 'geometric', parametric = 'gamma')
print(fout)

# posterior summary
fit <- fitted(fout)

# predictive density plot
plot(fit, ask = TRUE)

detach(faithful)

## End(Not run)
```

---

 bsaq

*Bayesian Shape-Restricted Spectral Analysis Quantile Regression*


---

## Description

This function fits a Bayesian semiparametric quantile regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors.

## Usage

```
bsaq(y, w, x, xmin, xmax, p, nbasis, nint, mcmc = list(), prior = list(),
    shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
    'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
    'IncreasingRotatedS', 'DecreasingRotatedS', 'InvertedU', 'Ushape'),
    marginal.likelihood = TRUE, spm.adequacy = FALSE)
```



**Arguments**

<code>y</code>	a vector of response values.
<code>w</code>	a vector or matrix giving covariates of dimension $n$ times $\text{ndimw}$ excluding intercept for a parametric component ( <code>w</code> can be omitted)
<code>x</code>	a vector or matrix giving covariates of dimension $n$ times $K$ for nonparametric components.
<code>xmin</code>	a vector or scalar giving user-specific minimum values of $x$ . The default values are minimum values of $x$ .
<code>xmax</code>	a vector or scalar giving user-specific maximum values of $x$ . The default values are maximum values of $x$ .
<code>p</code>	quantile of interest (default=0.5).
<code>nbasis</code>	number of cosine basis functions.
<code>nint</code>	number of grid points where the unknown function is evaluated for plotting. The default is 200.
<code>mcmc</code>	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): <code>nblow0</code> (1000) giving the number of initialization period for adaptive metropolis, <code>maxmodmet</code> (5) giving the maximum number of times to modify metropolis, <code>nblow</code> (10000) giving the number of MCMC in transition period, <code>nskip</code> (10) giving the thinning interval, <code>smcmc</code> (1000) giving the number of MCMC for analysis, and <code>ndisp</code> (1000) giving the number of saved draws to be displayed on screen (the function reports on the screen when every <code>ndisp</code> iterations have been carried out).
<code>prior</code>	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): <code>iflagprior</code> choosing a smoothing prior for spectral coefficients ( <code>iflagprior=0</code> assigns T-Smoother prior (default), <code>iflagprior=1</code> chooses Lasso-Smoother prior), <code>theta0_m0</code> and <code>theta0_s0</code> giving the hyperparameters for prior distribution of the spectral coefficients ( <code>theta0_m0</code> and <code>theta0_s0</code> are used when the functions have shape-restriction), <code>tau2_m0</code> , <code>tau2_s0</code> and <code>w0</code> giving the prior mean and standard deviation of smoothing prior (When <code>iflagprior=1</code> , <code>tau2_m0</code> is only used as the hyperparameter), <code>beta_m0</code> and <code>beta_v0</code> giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, <code>sigma2_m0</code> and <code>sigma2_v0</code> giving the prior mean and variance of the inverse gamma prior for the scale parameter of response, <code>alpha_m0</code> and <code>alpha_s0</code> giving the prior mean and standard deviation of the truncated normal prior distribution for the constant of integration, <code>iflagpsi</code> determining the prior of slope for logistic function in S or U shaped ( <code>iflagpsi=1</code> (default), slope $\psi$ is sampled and <code>iflagpsi=0</code> , $\psi$ is fixed), <code>psifixed</code> giving initial value ( <code>iflagpsi=1</code> ) or fixed value ( <code>iflagpsi=0</code> ) of slope, <code>omega_m0</code> and <code>omega_s0</code> giving the prior mean and standard deviation of the truncated normal prior distribution for the inflection point of S or U shaped function.
<code>shape</code>	a vector giving types of shape restriction.
<code>marginal.likelihood</code>	a logical variable indicating whether the log marginal likelihood is calculated. The methods of Gelfand and Dey (1994) and Newton and Raftery (1994) are used.
<code>spm.adequacy</code>	a logical variable indicating whether the log marginal likelihood of linear model is calculated. The marginal likelihood gives the values of the linear regression model excluding the nonlinear parts.

## Details

This generic function fits a Bayesian spectral analysis quantile regression model for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, the model assumed that the derivatives of the functions are squares of Gaussian processes.

Let  $y_i$  and  $w_i$  be the response and the vector of parametric predictors, respectively. Further, let  $x_{i,k}$  be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$y_i = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}) + \epsilon_i, \quad i = 1, \dots, n,$$

where  $f_k$  is an unknown shape-restricted function of the scalar  $x_{i,k} \in [0, 1]$  and the error terms  $\{\epsilon_i\}$  are a random sample from an asymmetric Laplace distribution,  $ALD_p(0, \sigma^2)$ , which has the following probability density function:

$$ALD_p(\epsilon; \mu, \sigma^2) = \frac{p(1-p)}{\sigma^2} \exp\left(-\frac{(x-\mu)[p-I(x \leq \mu)]}{\sigma^2}\right),$$

where  $0 < p < 1$  is the skew parameter,  $\sigma^2 > 0$  is the scale parameter,  $-\infty < \mu < \infty$  is the location parameter, and  $I(\cdot)$  is the indication function.

The prior of function without shape restriction is:

$$f(x) = Z(x),$$

where  $Z$  is a second-order Gaussian process with mean function equal to zero and covariance function  $\nu(s, t) = E[Z(s)Z(t)]$  for  $s, t \in [0, 1]$ . The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)$$

$$\varphi_0(x) = 1 \quad \text{and} \quad \varphi_j(x) = \sqrt{2} \cos(\pi j x), \quad j \geq 1, \quad 0 \leq x \leq 1$$

The shape-restricted functions are modeled by assuming the  $q$ th derivatives of  $f$  are squares of Gaussian processes:

$$f^{(q)}(x) = \delta Z^2(x) h(x), \quad \delta \in \{1, -1\}, \quad q \in \{1, 2\},$$

where  $h$  is the squish function. For monotonic, monotonic convex, and concave functions,  $h(x) = 1$ , while for S and U shaped functions,  $h$  is defined by

$$h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \quad \psi > 0, \quad 0 < \omega < 1$$

For the spectral coefficients of functions without shape constraints, the scale-invariant prior is used (The intercept is included in  $\beta$ ):

$$\theta_j | \sigma, \tau, \gamma \sim N(0, \sigma^2 \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

The priors for the spectral coefficients of shape restricted functions are:

$$\theta_0 | \sigma \sim N(m_{\theta_0}, \sigma v_{\theta_0}^2), \quad \theta_j | \sigma, \tau, \gamma \sim N(m_{\theta_j}, \sigma \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

To complete the model specification, the conjugate priors are assumed for  $\beta$  and  $\sigma$ :

$$\beta | \sigma \sim N(m_{0,\beta}, \sigma^2 V_{0,\beta}), \quad \sigma^2 \sim IG\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)$$

**Value**

An object of class `bsam` representing the Bayesian spectral analysis model fit. Generic functions such as `print`, `fitted` and `plot` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws`, the posterior samples of the fitted values are stored in the list `fit.draws`, and the MCMC samples for the log marginal likelihood are saved in the list `loglik.draws`. The output list also includes the following objects:

<code>post.est</code>	posterior estimates for all parameters in the model.
<code>lmarg.lm</code>	log marginal likelihood for linear quantile regression model.
<code>lmarg.gd</code>	log marginal likelihood using Gelfand-Dey method.
<code>lmarg.nr</code>	log marginal likelihood using Netwon-Raftery method, which is biased.
<code>rsquarey</code>	correlation between $y$ and $\hat{y}$ .
<code>call</code>	the matched call.
<code>mcmctime</code>	running time of Markov chain from <code>system.time()</code> .

**References**

Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, 27: 43-69.

Gelfand, A. E. and Dey, K. K. (1994) Bayesian model choice: asymptotics and exact calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, **56**, 501-514.

Kozumi, H. and Kobayashi, G. (2011) Gibbs sampling methods for Bayesian quantile regression. *Journal of Statistical Computation and Simulation*, **81**(11), 1565-1578.

Newton, M. A. and Raftery, A. E. (1994) Approximate Bayesian inference with the weighted likelihood bootstrap (with discussion). *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, **56**, 3-48.

**See Also**

[bsar](#), [gbsar](#)

**Examples**

```
## Not run:
#####
# Increasing-concave #
#####

# Simulate data
set.seed(1)

n <- 200
x <- runif(n)
y <- log(1 + 10*x) + rald(n, scale = 0.5, p = 0.5)

# Number of cosine basis functions
nbasis <- 50

# Fit the model with default priors and mcmc parameters
fout1 <- bsaq(y = y, x = x, p = 0.25, nbasis = nbasis,
```

```

        shape = 'IncreasingConcave')
fout2 <- bsaq(y = y, x = x, p = 0.5, nbasis = nbasis,
             shape = 'IncreasingConcave')
fout3 <- bsaq(y = y, x = x, p = 0.75, nbasis = nbasis,
             shape = 'IncreasingConcave')

# fitted values
fit1 <- fitted(fout1)
fit2 <- fitted(fout2)
fit3 <- fitted(fout3)

# plots
plot(x, y, lwd = 2, xlab = 'x', ylab = 'y')
lines(fit1$xgrid, fit1$wbeta$mean[1] + fit1$fxgrid$mean, lwd=2, col=2)
lines(fit2$xgrid, fit2$wbeta$mean[1] + fit2$fxgrid$mean, lwd=2, col=3)
lines(fit3$xgrid, fit3$wbeta$mean[1] + fit3$fxgrid$mean, lwd=2, col=4)
legend('topleft', legend = c('1st Quartile', '2nd Quartile', '3rd Quartile'),
      lwd = 2, col = 2:4, lty = 1)

## End(Not run)

```

bsaqdpm

*Bayesian Shape-Restricted Spectral Analysis Quantile Regression  
with Dirichlet Process Mixture Errors*

## Description

This function fits a Bayesian semiparametric quantile regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors. The model assumes that the errors follow a Dirichlet process mixture model.

## Usage

```

bsaqdpm(y, w, x, xmin, xmax, p, nbasis, nint,
        mcmc = list(), prior = list(), egrid, ngrid = 500,
        shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
                  'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
                  'IncreasingRotatedS', 'DecreasingRotatedS', 'InvertedU', 'Ushape'))

```

## Arguments

<code>y</code>	a vector of response values.
<code>w</code>	a vector or matrix giving covariates of dimension $n$ times $\text{ndimw}$ excluding intercept for a parametric component ( $w$ can be omitted)
<code>x</code>	a vector or matrix giving covariates of dimension $n$ times $K$ for nonparametric components.
<code>xmin</code>	a vector or scalar giving user-specific minimum values of $x$ . The default values are minimum values of $x$ .
<code>xmax</code>	a vector or scalar giving user-specific maximum values of $x$ . The default values are maximum values of $x$ .

p	quantile of interest (default=0.5).
nbasis	number of cosine basis functions.
nint	number of grid points where the unknown function is evaluated for plotting. The default is 200.
mcmc	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): nblow0 (1000) giving the number of initialization period for adaptive metropolis, maxmodmet (5) giving the maximum number of times to modify metropolis, nblow (10000) giving the number of MCMC in transition period, nskip (10) giving the thinning interval, smcmc (1000) giving the number of MCMC for analysis, and ndisp (1000) giving the number of saved draws to be displayed on screen (the function reports on the screen when every ndisp iterations have been carried out).
prior	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): iflagprior choosing a smoothing prior for spectral coefficients (iflagprior=0 assigns T-Smoother prior (default), iflagprior=1 chooses Lasso-Smoother prior), theta0_m0 and theta0_s0 giving the hyperparameters for prior distribution of the spectral coefficients (theta0_m0 and theta0_s0 are used when the functions have shape-restriction), tau2_m0, tau2_s0 and w0 giving the prior mean and standard deviation of smoothing prior (When iflagprior=1, tau2_m0 is only used as the hyperparameter), beta_m0 and beta_v0 giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, sigma2_m0 and sigma2_v0 giving the prior mean and variance of the inverse gamma prior for the scale parameter of response, alpha_m0 and alpha_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the constant of integration, iflagpsi determining the prior of slope for logistic function in S or U shaped (iflagpsi=1 (default), slope $\psi$ is sampled and iflagpsi=0, $\psi$ is fixed), psifixed giving initial value (iflagpsi=1) or fixed value (iflagpsi=0) of slope, omega_m0 and omega_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the inflection point of S or U shaped function.
egrid	a vector giving grid points where the residual density estimate is evaluated. The default range is from -10 to 10.
ngrid	a vector giving number of grid points where the residual density estimate is evaluated. The default value is 500.
shape	a vector giving types of shape restriction.

## Details

This generic function fits a Bayesian spectral analysis quantile regression model for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, the model assumes that the derivatives of the functions are squares of Gaussian processes. The model also assumes that the errors follow a Dirichlet process mixture model.

Let  $y_i$  and  $w_i$  be the response and the vector of parametric predictors, respectively. Further, let  $x_{i,k}$  be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$y_i = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}) + \epsilon_i, \quad i = 1, \dots, n,$$

where  $f_k$  is an unknown shape-restricted function of the scalar  $x_{i,k} \in [0, 1]$  and the error terms  $\{\epsilon_i\}$  are a random sample from a Dirichlet process mixture of an asymmetric Laplace distribution,  $ALD_p(0, \sigma^2)$ , which has the following probability density function:

$$\epsilon_i \sim f(\epsilon) = \int ALD_p(\epsilon; 0, \sigma^2) dG(\sigma^2),$$

$$G \sim DP(M, G_0), \quad G_0 = Ga\left(\sigma^{-2}; \frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right).$$

The prior of function without shape restriction is:

$$f(x) = Z(x),$$

where  $Z$  is a second-order Gaussian process with mean function equal to zero and covariance function  $\nu(s, t) = E[Z(s)Z(t)]$  for  $s, t \in [0, 1]$ . The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)$$

$$\varphi_0(x) = 1 \quad \text{and} \quad \varphi_j(x) = \sqrt{2} \cos(\pi j x), \quad j \geq 1, \quad 0 \leq x \leq 1$$

The shape-restricted functions are modeled by assuming the  $q$ th derivatives of  $f$  are squares of Gaussian processes:

$$f^{(q)}(x) = \delta Z^2(x) h(x), \quad \delta \in \{1, -1\}, \quad q \in \{1, 2\},$$

where  $h$  is the squish function. For monotonic, monotonic convex, and concave functions,  $h(x) = 1$ , while for S and U shaped functions,  $h$  is defined by

$$h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \quad \psi > 0, \quad 0 < \omega < 1$$

For the spectral coefficients of functions without shape constraints, the scale-invariant prior is used (The intercept is included in  $\beta$ ):

$$\theta_j | \tau, \gamma \sim N(0, \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

The priors for the spectral coefficients of shape restricted functions are:

$$\theta_0 \sim N(m_{\theta_0}, v_{\theta_0}^2), \quad \theta_j | \tau, \gamma \sim N(m_{\theta_j}, \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

To complete the model specification, the popular normal prior is assumed for  $\beta$ :

$$\beta | \sim N(m_{0,\beta}, V_{0,\beta})$$

## Value

An object of class `bsam` representing the Bayesian spectral analysis model fit. Generic functions such as `print`, `fitted` and `plot` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws`, the posterior samples of the fitted values are stored in the list `fit.draws`, and the MCMC samples for the log marginal likelihood are saved in the list `loglik.draws`. The output list also includes the following objects:

post.est	posterior estimates for all parameters in the model.
lpm1	log pseudo marginal likelihood using Mukhopadhyay and Gelfand method.
rsquarey	correlation between $y$ and $\hat{y}$ .
imodmet	the number of times to modify Metropolis.
pmet	proportion of $\theta$ accepted after burn-in.
call	the matched call.
mcmctime	running time of Markov chain from <code>system.time()</code> .

## References

- Kozumi, H. and Kobayashi, G. (2011) Gibbs sampling methods for Bayesian quantile regression. *Journal of Statistical Computation and Simulation*, **81**(11), 1565-1578.
- Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, **27**, 43-69.
- MacEachern, S. N. and Muller, P. (1998) Estimating mixture of Dirichlet process models. *Journal of Computational and Graphical Statistics*, **7**, 223-238.
- Mukhopadhyay, S. and Gelfand, A. E. (1997) Dirichlet process mixed generalized linear models. *Journal of the American Statistical Association*, **92**, 633-639.
- Neal, R. M. (2000) Markov chain sampling methods for Dirichlet process mixture models. *Journal of Computational and Graphical Statistics*, **9**, 249-265.

## See Also

[bsaq](#), [bsardpm](#)

## Examples

```
## Not run:
#####
# Increasing-concave #
#####

# Simulate data
set.seed(1)

n <- 500
x <- runif(n)
e <- c(rald(n/2, scale = 0.5, p = 0.5),
      rald(n/2, scale = 3, p = 0.5))
y <- log(1 + 10*x) + e

# Number of cosine basis functions
nbasis <- 50

# Fit the model with default priors and mcmc parameters
fout1 <- bsaqdpm(y = y, x = x, p = 0.25, nbasis = nbasis,
                shape = 'IncreasingConcave')
fout2 <- bsaqdpm(y = y, x = x, p = 0.5, nbasis = nbasis,
                shape = 'IncreasingConcave')
fout3 <- bsaqdpm(y = y, x = x, p = 0.75, nbasis = nbasis,
                shape = 'IncreasingConcave')
```

```

# fitted values
fit1 <- fitted(fout1)
fit2 <- fitted(fout2)
fit3 <- fitted(fout3)

# plots
plot(x, y, lwd = 2, xlab = 'x', ylab = 'y')
lines(fit1$xgrid, fit1$wbeta$mean[1] + fit1$fxgrid$mean, lwd=2, col=2)
lines(fit2$xgrid, fit2$wbeta$mean[1] + fit2$fxgrid$mean, lwd=2, col=3)
lines(fit3$xgrid, fit3$wbeta$mean[1] + fit3$fxgrid$mean, lwd=2, col=4)
legend('topleft', legend=c('1st Quartile', '2nd Quartile', '3rd Quartile'),
      lwd=2, col=2:4, lty=1)

## End(Not run)

```

---

bsar

*Bayesian Shape-Restricted Spectral Analysis Regression*


---

## Description

This function fits a Bayesian semiparametric regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors.

## Usage

```

bsar(y, w, x, xmin, xmax, nbasis, nint, mcmc = list(), prior = list(),
     shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
               'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
               'IncreasingRotatedS', 'DecreasingRotatedS', 'InvertedU', 'Ushape'),
     marginal.likelihood = TRUE, spm.adequacy = FALSE)

```

## Arguments

y	a vector of response values.
w	a vector or matrix giving covariates of dimension n times ndimw excluding intercept for a parametric component (w can be omitted)
x	a vector or matrix giving covariates of dimension n times K for nonparametric components.
xmin	a vector or scalar giving user-specific minimum values of x. The default values are minimum values of x.
xmax	a vector or scalar giving user-specific maximum values of x. The default values are maximum values of x.
nbasis	number of cosine basis functions.
nint	number of grid points where the unknown function is evaluated for plotting. The default is 200.



mcmc	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): nblow0 (1000) giving the number of initialization period for adaptive metropolis, maxmodmet (5) giving the maximum number of times to modify metropolis, nblow (10000) giving the number of MCMC in transition period, nskip (10) giving the thinning interval, smcmc (1000) giving the number of MCMC for analysis, and ndisp (1000) giving the number of saved draws to be displayed on screen (the function reports on the screen when every ndisp iterations have been carried out).
prior	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): iflagprior choosing a smoothing prior for spectral coefficients (iflagprior=0 assigns T-Smoother prior (default), iflagprior=1 chooses Lasso-Smoother prior), theta0_m0 and theta0_s0 giving the hyperparameters for prior distribution of the spectral coefficients (theta0_m0 and theta0_s0 are used when the functions have shape-restriction), tau2_m0, tau2_s0 and w0 giving the prior mean and standard deviation of smoothing prior (When iflagprior=1, tau2_m0 is only used as the hyperparameter), beta_m0 and beta_v0 giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, sigma2_m0 and sigma2_v0 giving the prior mean and variance of the inverse gamma prior for the scale parameter of response, alpha_m0 and alpha_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the constant of integration, iflagpsi determining the prior of slope for logisitic function in S or U shaped (iflagpsi=1 (default), slope $\psi$ is sampled and iflagpsi=0, $\psi$ is fixed), psifixed giving initial value (iflagpsi=1) or fixed value (iflagpsi=0) of slope, omega_m0 and omega_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the inflection point of S or U shaped function.
shape	a vector giving types of shape restriction.
marginal.likelihood	a logical variable indicating whether the log marginal likelihood is calculated. The methods of Gelfand and Dey (1994) and Newton and Raftery (1994) are used.
spm.adequacy	a logical variable indicating whether the log marginal likelihood of linear model is calculated. The marginal likelihood gives the values of the linear regression model excluding the nonlinear parts.

## Details

This generic function fits a Bayesian spectral analysis regression model (Lenk and Choi, 2015) for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, they assumed that the derivatives of the functions are squares of Gaussian processes.

Let  $y_i$  and  $w_i$  be the response and the vector of parametric predictors, respectively. Further, let  $x_{i,k}$  be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$y_i = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}) + \epsilon_i, \quad i = 1, \dots, n,$$

where  $f_k$  is an unknown shape-restricted function of the scalar  $x_{i,k} \in [0, 1]$  and the error terms  $\{\epsilon_i\}$  are a random sample from a normal distribution,  $N(0, \sigma^2)$ .

The prior of function without shape restriction is:

$$f(x) = Z(x),$$

where  $Z$  is a second-order Gaussian process with mean function equal to zero and covariance function  $\nu(s, t) = E[Z(s)Z(t)]$  for  $s, t \in [0, 1]$ . The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)$$

$$\varphi_0(x) = 1 \text{ and } \varphi_j(x) = \sqrt{2} \cos(\pi j x), \quad j \geq 1, \quad 0 \leq x \leq 1$$

The shape-restricted functions are modeled by assuming the  $q$ th derivatives of  $f$  are squares of Gaussian processes:

$$f^{(q)}(x) = \delta Z^2(x) h(x), \quad \delta \in \{1, -1\}, \quad q \in \{1, 2\},$$

where  $h$  is the squish function. For monotonic, monotonic convex, and concave functions,  $h(x) = 1$ , while for S and U shaped functions,  $h$  is defined by

$$h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \quad \psi > 0, \quad 0 < \omega < 1$$

For the spectral coefficients of functions without shape constraints, the scale-invariant prior is used (The intercept is included in  $\beta$ ):

$$\theta_j | \sigma, \tau, \gamma \sim N(0, \sigma^2 \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

The priors for the spectral coefficients of shape restricted functions are:

$$\theta_0 | \sigma \sim N(m_{\theta_0}, \sigma v_{\theta_0}^2), \quad \theta_j | \sigma, \tau, \gamma \sim N(m_{\theta_j}, \sigma \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

To complete the model specification, the conjugate priors are assumed for  $\beta$  and  $\sigma$ :

$$\beta | \sigma \sim N(m_{0,\beta}, \sigma^2 V_{0,\beta}), \quad \sigma^2 \sim IG\left(\frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right)$$

## Value

An object of class `bsam` representing the Bayesian spectral analysis model fit. Generic functions such as `print`, `fitted` and `plot` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws`, the posterior samples of the fitted values are stored in the list `fit.draws`, and the MCMC samples for the log marginal likelihood are saved in the list `loglik.draws`. The output list also includes the following objects:

<code>post.est</code>	posterior estimates for all parameters in the model.
<code>lmarg.lm</code>	log marginal likelihood for linear regression model.
<code>lmarg.gd</code>	log marginal likelihood using Gelfand-Dey method.
<code>lmarg.nr</code>	log marginal likelihood using Netwon-Raftery method, which is biased.
<code>rsquarey</code>	correlation between $y$ and $\hat{y}$ .
<code>call</code>	the matched call.
<code>mcmctime</code>	running time of Markov chain from <code>system.time()</code> .

## References

Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, **27**, 43-69.

Gelfand, A. E. and Dey, K. K. (1994) Bayesian model choice: asymptotics and exact calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, **56**, 501-514.

Newton, M. A. and Raftery, A. E. (1994) Approximate Bayesian inference with the weighted likelihood bootstrap (with discussion). *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, **56**, 3-48.

## See Also

[bsardpm](#)

## Examples

```
## Not run:

#####
# Increasing Convex to Concave (S-shape) #
#####

# simulate data
f <- function(x) 5*exp(-10*(x - 1)^4) + 5*x^2

set.seed(1)

n <- 100
x <- runif(n)
y <- f(x) + rnorm(n, sd = 1)

# Number of cosine basis functions
nbasis <- 50

# Fit the model with default priors and mcmc parameters
fout <- bsar(y = y, x = x, nbasis = nbasis, shape = 'IncreasingConvex',
             spm.adequacy = TRUE)

# Summary
print(fout)

# Trace plots
plot(fout)

# fitted values
fit <- fitted(fout)

# Plot
plot(fit)

#####
# Additive Model #
# Monotone-Increasing and Increasing-Convex #
#####

# Simulate data
```

```

f1 <- 2*pi*x + sin(2*pi*x)
f2 <- exp(6*x - 3)

n <- 200
x1 <- runif(n)
x2 <- runif(n)
x <- cbind(x1, x2)

y <- 5 + f1(x1) + f2(x2) + rnorm(n, sd = 0.5)

# Number of cosine basis functions
nbasis <- 50

# MCMC parameters
mcmc <- list(nblow0 = 1000, nblow = 10000, nskip = 10,
            smcmc = 5000, ndisp = 1000, maxmodmet = 10)

# Prior information
xmin <- apply(x, 2, min)
xmax <- apply(x, 2, max)
xrange <- xmax - xmin
prior <- list(iflagprior = 0, theta0_m0 = 0, theta0_s0 = 100,
            tau2_m0 = 1, tau2_v0 = 100, w0 = 2,
            beta_m0 = numeric(nparw), beta_v0 = diag(100, nparw),
            sigma2_m0 = 1, sigma2_v0 = 1000,
            alpha_m0 = 3, alpha_s0 = 50, iflagpsi = 1,
            psifixed = 1000, omega_m0 = (xmin + xmax)/2,
            omega_s0 = (xrange)/8)

# Fit the model with user specific priors and mcmc parameters
fout <- bsar(y = y, x = x, nbasis = nbasis, mcmc = mcmc, prior = prior,
            shape = c('Increasing', 'IncreasingS'))

# Summary
print(fout)

## End(Not run)

```

---

bsardpm

*Bayesian Shape-Restricted Spectral Analysis Regression with Dirichlet Process Mixture Errors*


---

## Description

This function fits a Bayesian semiparametric regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors. The model assumes that the errors follow a Dirichlet process mixture model.

## Usage

```

bsardpm(y, w, x, xmin, xmax, nbasis, nint,
mcmc = list(), prior = list(), egrid, ngrid, location = TRUE,

```

```
shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
'IncreasingRotatedS', 'DecreasingRotatedS', 'InvertedU', 'Ushape'))
```

### Arguments

y	a vector of response values.
w	a vector or matrix giving covariates of dimension n times ndimw excluding intercept for a parametric component (w can be omitted)
x	a vector or matrix giving covariates of dimension n times K for nonparametric components.
xmin	a vector or scalar giving user-specific minimum values of x. The default values are minimum values of x.
xmax	a vector or scalar giving user-specific maximum values of x. The default values are maximum values of x.
nbasis	number of cosine basis functions.
nint	number of grid points where the unknown function is evaluated for plotting. The default is 200.
mcmc	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): nblow0 (1000) giving the number of initialization period for adaptive metropolis, maxmodmet (5) giving the maximum number of times to modify metropolis, nblow (10000) giving the number of MCMC in transition period, nskip (10) giving the thinning interval, smcmc (1000) giving the number of MCMC for analysis, and ndisp (1000) giving the number of saved draws to be displayed on screen (the function reports on the screen when every ndisp iterations have been carried out).
prior	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): iflagprior choosing a smoothing prior for spectral coefficients (iflagprior=0 assigns T-Smoother prior (default), iflagprior=1 chooses Lasso-Smoother prior), theta0_m0 and theta0_s0 giving the hyperparameters for prior distribution of the spectral coefficients (theta0_m0 and theta0_s0 are used when the functions have shape-restriction), tau2_m0, tau2_s0 and w0 giving the prior mean and standard deviation of smoothing prior (When iflagprior=1, tau2_m0 is only used as the hyperparameter), beta_m0 and beta_v0 giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, sigma2_m0 and sigma2_v0 giving the prior mean and variance of the inverse gamma prior for the scale parameter of response, alpha_m0 and alpha_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the constant of integration, iflagpsi determining the prior of slope for logisitic function in S or U shaped (iflagpsi=1 (default), slope $\psi$ is sampled and iflagpsi=0, $\psi$ is fixed), psifixed giving initial value (iflagpsi=1) or fixed value (iflagpsi=0) of slope, omega_m0 and omega_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the inflection point of S or U shaped function.
egrid	a vector giving grid points where the residual density estimate is evaluated. The default range is from -10 to 10.
ngrid	a vector giving number of grid points where the residual density estimate is evaluated. The default value is 500.

location	a logical value. If it is true, error density is modelled using location-scale mixture.
shape	a vector giving types of shape restriction.

### Details

This generic function fits a Bayesian spectral analysis regression model for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, the model assumes that the derivatives of the functions are squares of Gaussian processes. The model also assumes that the errors follow a Dirichlet process mixture model.

Let  $y_i$  and  $w_i$  be the response and the vector of parametric predictors, respectively. Further, let  $x_{i,k}$  be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$y_i = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}) + \epsilon_i, \quad i = 1, \dots, n,$$

where  $f_k$  is an unknown shape-restricted function of the scalar  $x_{i,k} \in [0, 1]$  and the error terms  $\{\epsilon_i\}$  are a random sample from a Dirichlet process mixture model,

1. scale mixture :

$$\begin{aligned} \epsilon_i &\sim f(\epsilon) = \int N(\epsilon; 0, \sigma^2) dG(\sigma^2), \\ G &\sim DP(M, G_0), \quad G_0 = Ga\left(\sigma^{-2}; \frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right). \end{aligned}$$

2. location-scale mixture :

$$\begin{aligned} \epsilon_i &\sim f(\epsilon) = \int N(\epsilon; \mu, \sigma^2) dG(\mu, \sigma^2), \\ G &\sim DP(M, G_0), \quad G_0 = N(\mu; \mu_0, \kappa\sigma^2) Ga\left(\sigma^{-2}; \frac{r_{0,\sigma}}{2}, \frac{s_{0,\sigma}}{2}\right). \end{aligned}$$

The prior of function without shape restriction is:

$$f(x) = Z(x),$$

where  $Z$  is a second-order Gaussian process with mean function equal to zero and covariance function  $\nu(s, t) = E[Z(s)Z(t)]$  for  $s, t \in [0, 1]$ . The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)$$

$$\varphi_0(x) = 1 \quad \text{and} \quad \varphi_j(x) = \sqrt{2} \cos(\pi j x), \quad j \geq 1, \quad 0 \leq x \leq 1$$

The shape-restricted functions are modeled by assuming the  $q$ th derivatives of  $f$  are squares of Gaussian processes:

$$f^{(q)}(x) = \delta Z^2(x) h(x), \quad \delta \in \{1, -1\}, \quad q \in \{1, 2\},$$

where  $h$  is the squish function. For monotonic, monotonic convex, and concave functions,  $h(x) = 1$ , while for S and U shaped functions,  $h$  is defined by

$$h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \quad \psi > 0, \quad 0 < \omega < 1$$

For the spectral coefficients of functions without shape constraints, the scale-invariant prior is used (The intercept is included in  $\beta$ ):

$$\theta_j | \tau, \gamma \sim N(0, \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

The priors for the spectral coefficients of shape restricted functions are:

$$\theta_0 \sim N(m_{\theta_0}, v_{\theta_0}^2), \quad \theta_j | \tau, \gamma \sim N(m_{\theta_j}, \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

To complete the model specification, the popular normal prior is assumed for  $\beta$ :

$$\beta | \sim N(m_{0,\beta}, V_{0,\beta})$$

## Value

An object of class `bsam` representing the Bayesian spectral analysis model fit. Generic functions such as `print`, `fitted` and `plot` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws`, the posterior samples of the fitted values are stored in the list `fit.draws`, and the MCMC samples for the log marginal likelihood are saved in the list `loglik.draws`. The output list also includes the following objects:

<code>post.est</code>	posterior estimates for all parameters in the model.
<code>lpml</code>	log pseudo marginal likelihood using Mukhopadhyay and Gelfand method.
<code>imodmet</code>	the number of times to modify Metropolis.
<code>pmet</code>	proportion of $\theta$ accepted after burn-in.
<code>call</code>	the matched call.
<code>mcmctime</code>	running time of Markov chain from <code>system.time()</code> .

## References

- Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, **27**, 43-69.
- MacEachern, S. N. and Muller, P. (1998) Estimating mixture of Dirichlet process models. *Journal of Computational and Graphical Statistics*, **7**, 223-238.
- Mukhopadhyay, S. and Gelfand, A. E. (1997) Dirichlet process mixed generalized linear models. *Journal of the American Statistical Association*, **92**, 633-639.
- Neal, R. M. (2000) Markov chain sampling methods for Dirichlet process mixture models. *Journal of Computational and Graphical Statistics*, **9**, 249-265.

## See Also

[bsar](#), [bsaqdpm](#)

## Examples

```
## Not run:
#####
# Increasing-convex #
#####

# Simulate data
```

```

set.seed(1)

n <- 200
x <- runif(n)
e <- c(rnorm(n/2, sd = 0.5), rnorm(n/2, sd = 3))
y <- exp(6*x - 3) + e

# Number of cosine basis functions
nbasis <- 50

# Fit the model with default priors and mcmc parameters
fout <- bsardpm(y = y, x = x, nbasis = nbasis, shape = 'IncreasingConvex')

# Summary
print(fout)

# fitted values
fit <- fitted(fout)

# Plot
plot(fit, ask = TRUE)

## End(Not run)

```

---

Elec.demand

*Electricity demand data*

---

### Description

The Elec.demand data consists of 288 quarterly observations in Ontario from 1971 to 1994.

### Usage

```
data(Elec.demand)
```

### Format

A data frame with 288 observations on the following 7 variables.

**quarter** date (yyyy-mm) from 1971 to 1994

**enerm** electricity demand.

**gdp** gross domestic product.

**pelec** price of electricity.

**pgas** price of natural gas.

**hddqm** the number of heating degree days relative to a reference temperature.

**cddqm** the number of cooling degree days relative to a reference temperature.

### Source

Yatchew, A. (2003). *Semiparametric Regression for the Applied Econometrician*. Cambridge University Press.



## References

Engle, R. F., Granger, C. W. J., Rice, J. and Weiss, A. (1986). Semiparametric estimates of the relation between weather and electricity sales. *Journal of the American Statistical Association*, **81**, 310-320.

Lenk, P. and Choi, T. (2017). Bayesian analysis of shape-restricted functions using Gaussian process priors. *Statistica Sinica*, **27**, 43-69.

## Examples

```
## Not run:
data(Elec.demand)
plot(Elec.demand)

## End(Not run)
```

---

fitted.blm

*Compute fitted values for a blm object*

---

## Description

Computes pointwise posterior means and 95% credible intervals of the fitted Bayesian linear models.

## Usage

```
## S3 method for class 'blm'
fitted(object, alpha = 0.05, HPD = TRUE, ...)
```

## Arguments

object	a bsam object
alpha	a numeric scalar in the interval (0,1) giving the $100(1 - \alpha)\%$ credible intervals.
HPD	a logical variable indicating whether the $100(1 - \alpha)\%$ Highest Posterior Density (HPD) intervals are calculated. If HPD=FALSE, the $100(1 - \alpha)\%$ equal-tail credible intervals are calculated. The default is TRUE.
...	not used

## Details

None.

## Value

A list containing posterior means and 95% credible intervals.

The output list includes the following object:

wbeta	posterior estimates for regression function.
-------	--

**References**

Chen, M., Shao, Q. and Ibrahim, J. (2000) *Monte Carlo Methods in Bayesian computation*. Springer-Verlag New York, Inc.

**See Also**

[blq](#), [blr](#)

**Examples**

```
## See examples for blq and blr
```

---

fitted.bsad

*Compute fitted values for a bsad object*

---

**Description**

Computes pointwise posterior means and  $100(1 - \alpha)\%$  credible intervals of the fitted Bayesian spectral analysis density estimation model.

**Usage**

```
## S3 method for class 'bsad'
fitted(object, alpha = 0.05, HPD = TRUE, ...)
```

**Arguments**

object	a bsad object
alpha	a numeric scalar in the interval (0,1) giving the $100(1 - \alpha)\%$ credible intervals.
HPD	a logical variable indicating whether the $100(1 - \alpha)\%$ Highest Posterior Density (HPD) intervals are calculated. If HPD=FALSE, the $100(1 - \alpha)\%$ equal-tail credible intervals are calculated. The default is TRUE.
...	not used

**Details**

None.

**Value**

A list object of class `fitted.bsad` containing posterior means and  $100(1 - \alpha)\%$  credible intervals. Generic function `plot` displays the results of the fit.

The output list includes the following objects:

fpar	posterior estimates for parametric model.
fsemi	posterior estimates for semiparametric model.
fsemiMaxKappa	posterior estimates for semiparametric model with maximum number of basis.

**See Also**[bsad](#)**Examples**

```
## See examples for bsad
```

---

fitted.bsam	<i>Compute fitted values for a bsam object</i>
-------------	--

---

**Description**

Computes pointwise posterior means and  $100(1 - \alpha)\%$  credible intervals of the fitted Bayesian spectral analysis models.

**Usage**

```
## S3 method for class 'bsam'
fitted(object, alpha = 0.05, HPD = TRUE, ...)
```

**Arguments**

object	a bsam object
alpha	a numeric scalar in the interval (0,1) giving the $100(1 - \alpha)\%$ credible intervals.
HPD	a logical variable indicating whether the $100(1 - \alpha)\%$ Highest Posterior Density (HPD) intervals are calculated. If HPD=FALSE, the $100(1 - \alpha)\%$ equal-tail credible intervals are calculated. The default is TRUE.
...	not used

**Details**

None.

**Value**

A list object of class `fitted.bsam` containing posterior means and  $100(1 - \alpha)\%$  credible intervals. Generic function `plot` displays the results of the fit.

The output list includes the following objects:

fxobs	posterior estimates for unknown functions over observation.
fxgrid	posterior estimates for unknown functions over grid points.
wbeta	posterior estimates for parametric part.
yhat	posterior estimates for fitted values of response. For <a href="#">gbsar</a> , it gives posterior estimates for expectation of response.

**See Also**

[bsaq](#), [bsaqdpm](#), [bsar](#), [bsardpm](#)

**Examples**

```
## See examples for bsaq, bsaqdp, bsar, and bsardpm
```

---

fitted.bsamdpm	<i>Compute fitted values for a bsamdpm object</i>
----------------	---

---

**Description**

Computes pointwise posterior means and  $100(1 - \alpha)\%$  credible intervals of the fitted Bayesian spectral analysis models with Dirichlet process mixture error.

**Usage**

```
## S3 method for class 'bsamdpm'
fitted(object, alpha = 0.05, HPD = TRUE, ...)
```

**Arguments**

object	a bsamdpm object
alpha	a numeric scalar in the interval (0,1) giving the $100(1 - \alpha)\%$ credible intervals.
HPD	a logical variable indicating whether the $100(1 - \alpha)\%$ Highest Posterior Density (HPD) intervals are calculated. If HPD=FALSE, the $100(1 - \alpha)\%$ equal-tail credible intervals are calculated. The default is TRUE.
...	not used

**Details**

None.

**Value**

A list object of class fitted.bsamdpm containing posterior means and 95% credible intervals. Generic function plot displays the results of the fit.

The output list includes the following objects:

edens	posterior estimate for unknown error distribution over grid points.
fxobs	posterior estimates for unknown functions over observation.
fxgrid	posterior estimates for unknown functions over grid points.
wbeta	posterior estimates for parametric part.
yhat	posterior estimates for fitted values of response.

**See Also**

[bsaqdp](#), [bsardpm](#)

**Examples**

```
## See examples for bsaqdp and bsardpm
```

---

function.shape	<i>Shape of Nonlinear Function</i>
----------------	------------------------------------

---

**Description**

This function returns types of functions with shape constraints.

**Usage**

```
function.shape(shape = c("Free", "Increasing", "Decreasing",
  "IncreasingConvex", "DecreasingConcave", "IncreasingConcave",
  "DecreasingConvex", "IncreasingS", "DecreasingS",
  "IncreasingRotatedS", "DecreasingRotatedS", "InvertedU", "Ushape"))
```

**Arguments**

shape                    a vector giving types of shape restriction.

**Note**

This function is called by bsaq, bsaqdpn, bsar, bsardpn, and gbsar, not called by user.

---

gblr	<i>Generalized Bayesian Linear Models</i>
------	---

---

**Description**

This function fits a Bayesian generalized linear regression model.

**Usage**

```
gblr(y, w, n, family, link, mcmc = list(), prior = list(), marginal.likelihood = TRUE)
```

**Arguments**

y	a vector or matrix of response values: The multinomial model uses the categorical indicator responses (matrix).
w	a vector or matrix giving covariates of dimension n times ndimw excluding intercept for a parametric component.
n	an integer vector containing the number of trials for binomial data.
family	a description of the error distribution to be used in the model: The families contains bernoulli ("bernoulli"), poisson ("poisson"), negative-binomial ("negative.binomial"), poisson-gamma mixture ("poisson.gamma").
link	a description of the link function to be used in the model.

mcmc	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): <code>nblow</code> (10000) giving the number of MCMC in transition period, <code>nskip</code> (10) giving the thinning interval, <code>smcmc</code> (1000) giving the number of MCMC for analysis, and <code>ndisp</code> (1000) giving the number of saved draws to be displayed on screen (the function reports on the screen when every <code>ndisp</code> iterations have been carried out).
prior	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): <code>beta_m0</code> and <code>beta_v0</code> giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, <code>kappa_m0</code> and <code>kappa_v0</code> giving the prior mean and variance of the gamma prior distribution for dispersion parameter (negative-binomial).
marginal.likelihood	a logical variable indicating whether the log marginal likelihood is calculated. The methods of Gelfand and Dey (1994) is used.

### Details

This generic function fits a Bayesian generalized linear regression models.

Let  $y_i$  and  $w_i$  be the response and the vector of parametric predictors, respectively. The model is as follows.

$$y_i | \mu_i \sim F(\mu_i),$$

$$g(\mu_i) = w_i^T \beta, \quad i = 1, \dots, n,$$

where  $g(\cdot)$  is a link function and  $F(\cdot)$  is a distribution of an exponential family.

For unknown coefficients, the following prior is assumed for  $\beta$ :

$$\beta \sim N(m_{0,\beta}, V_{0,\beta})$$

The prior for the dispersion parameter of negative-binomial regression is

$$\kappa \sim Ga(r_0, s_0)$$

### Value

An object of class `blm` representing the generalized Bayesian linear model fit. Generic functions such as `print`, `fitted` and `plot` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws`, the posterior samples of the fitted values are stored in the list `fit.draws`, and the MCMC samples for the log marginal likelihood are saved in the list `loglik.draws`. The output list also includes the following objects:

<code>post.est</code>	posterior estimates for all parameters in the model.
<code>lmarg</code>	log marginal likelihood using Gelfand-Dey method.
<code>family</code>	the family object used.
<code>link</code>	the link object used.
<code>methods</code>	the method object used in the logit model.
<code>call</code>	the matched call.
<code>mcmctime</code>	running time of Markov chain from <code>system.time()</code> .

## References

Albert, J. H. and Chib, S. (1993) Bayesian Analysis of Binary and Polychotomous Response Data. *Journal of the American Statistical Association*, **88**, 669-679.

Holmes, C. C. and Held, L. (2006) Bayesian Auxiliary Variables Models for Binary and Multinomial Regression. *Bayesian Analysis*, **1**, 145-168.

Gelfand, A. E. and Dey, K. K. (1994) Bayesian model choice: asymptotics and exact calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, **56**, 501-514.

## See Also

[blr](#), [blq](#)

## Examples

```
## Not run:
#####
# Poisson Regression Model #
#####

# Simulate data
set.seed(1)

n <- 100
x <- runif(n)
y <- rpois(n, exp(0.5 + x*0.4))

# Fit the model with default priors and mcmc parameters
fout <- gblr(y = y, w = x, family = 'poisson', link = 'log')

# Summary
print(fout)

# Plot
plot(fout)

# fitted values
fitf <- itted(fout)

## End(Not run)
```

---

gbsar

*Bayesian Shape-Restricted Spectral Analysis for Generalized Partial Linear Models*

---

## Description

This function fits a Bayesian generalized partial linear regression model to estimate shape-restricted functions using a spectral analysis of Gaussian process priors.

**Usage**

```
gbsar(y, w, x, xmin, xmax, n, family, link, nbasis, nint, mcmc = list(), prior = list(),
      shape = c('Free', 'Increasing', 'Decreasing', 'IncreasingConvex', 'DecreasingConcave',
                'IncreasingConcave', 'DecreasingConvex', 'IncreasingS', 'DecreasingS',
                'IncreasingRotatedS', 'DecreasingRotatedS', 'InvertedU', 'Ushape'),
      marginal.likelihood = TRUE)
```

**Arguments**

<code>y</code>	a vector of response values.
<code>w</code>	a vector or matrix giving covariates of dimension $n$ times $\text{ndimw}$ excluding intercept for a parametric component ( $w$ can be omitted)
<code>x</code>	a vector or matrix giving covariates of dimension $n$ times $K$ for nonparametric components.
<code>xmin</code>	a vector or scalar giving user-specific minimum values of $x$ . The default values are minimum values of $x$ .
<code>xmax</code>	a vector or scalar giving user-specific maximum values of $x$ . The default values are maximum values of $x$ .
<code>n</code>	an integer vector containing the number of trials for binomial data.
<code>family</code>	a description of the error distribution to be used in the model: The families contains bernoulli (“bernoulli”), poisson (“poisson”), negative-binomial (“negative.binomial”), poisson-gamma mixture (“poisson.gamma”).
<code>link</code>	a description of the link function to be used in the model.
<code>nbasis</code>	number of cosine basis functions.
<code>nint</code>	number of grid points where the unknown function is evaluated for plotting. The default is 200.
<code>mcmc</code>	a list giving the MCMC parameters. The list includes the following integers (with default values in parentheses): <code>nblow0</code> (1000) giving the number of initialization period for adaptive metropolis, <code>maxmodmet</code> (5) giving the maximum number of times to modify metropolis, <code>nblow</code> (10000) giving the number of MCMC in transition period, <code>nskip</code> (10) giving the thinning interval, <code>smcmc</code> (1000) giving the number of MCMC for analysis, and <code>ndisp</code> (1000) giving the number of saved draws to be displayed on screen (the function reports on the screen when every <code>ndisp</code> iterations have been carried out).
<code>prior</code>	a list giving the prior information. The list includes the following parameters (default values specify the non-informative prior): <code>iflagprior</code> choosing a smoothing prior for spectral coefficients ( <code>iflagprior=0</code> assigns T-Smoothing prior (default), <code>iflagprior=1</code> chooses Lasso-Smoothing prior), <code>theta_m0</code> , <code>theta0_m0</code> and <code>theta0_s0</code> giving the hyperparameters for prior distribution of the spectral coefficients ( <code>theta0_m0</code> and <code>theta0_s0</code> are used when the functions have shape-restriction), <code>tau2_m0</code> , <code>tau2_s0</code> and <code>w0</code> giving the prior mean and standard deviation of smoothing prior (When <code>iflagprior=1</code> , <code>tau2_m0</code> is only used as the hyperparameter), <code>beta_m0</code> and <code>beta_v0</code> giving the hyperparameters of the multivariate normal distribution for parametric part including intercept, <code>alpha_m0</code> and <code>alpha_s0</code> giving the prior mean and standard deviation of the truncated normal prior distribution for the constant of integration, <code>iflagpsi</code> determining the prior of slope for logistic function in S or U shaped ( <code>iflagpsi=1</code> (default), slope $\psi$ is sampled and <code>iflagpsi=0</code> , $\psi$ is fixed), <code>psifixed</code> giving initial value



	(iflagpsi=1) or fixed value (iflagpsi=0) of slope, omega_m0 and omega_s0 giving the prior mean and standard deviation of the truncated normal prior distribution for the inflection point of S or U shaped function, kappa_m0 and kappa_v0 giving the prior mean and variance of the gammal prior distribution for dispersion parameter (negative-binomial).
shape	a vector giving types of shape restriction.
marginal.likelihood	a logical variable indicating whether the log marginal likelihood is calculated. The methods of Gelfand and Dey (1994) and Newton and Raftery (1994) are used.

## Details

This generic function fits a Bayesian generalized partial linear regression models for estimating shape-restricted functions using Gaussian process priors. For enforcing shape-restrictions, they assumed that the derivatives of the functions are squares of Gaussian processes.

Let  $y_i$  and  $w_i$  be the response and the vector of parametric predictors, respectively. Further, let  $x_{i,k}$  be the covariate related to the response through an unknown shape-restricted function. The model for estimating shape-restricted functions is as follows.

$$y_i | \mu_i \sim F(\mu_i),$$

$$g(\mu_i) = w_i^T \beta + \sum_{k=1}^K f_k(x_{i,k}), \quad i = 1, \dots, n,$$

where  $g(\cdot)$  is a link function and  $f_k$  is an unknown nonlinear function of the scalar  $x_{i,k} \in [0, 1]$ . The prior of function without shape restriction is:

$$f(x) = Z(x),$$

where  $Z$  is a second-order Gaussian process with mean function equal to zero and covariance function  $\nu(s, t) = E[Z(s)Z(t)]$  for  $s, t \in [0, 1]$ . The Gaussian process is expressed with the spectral representation based on cosine basis functions:

$$Z(x) = \sum_{j=0}^{\infty} \theta_j \varphi_j(x)$$

$$\varphi_0(x) = 1 \quad \text{and} \quad \varphi_j(x) = \sqrt{2} \cos(\pi j x), \quad j \geq 1, \quad 0 \leq x \leq 1$$

The shape-restricted functions are modeled by assuming the  $q$ th derivatives of  $f$  are squares of Gaussian processes:

$$f^{(q)}(x) = \delta Z^2(x) h(x), \quad \delta \in \{1, -1\}, \quad q \in \{1, 2\},$$

where  $h$  is the squish function. For monotonic, monotonic convex, and concave functions,  $h(x) = 1$ , while for S and U shaped functions,  $h$  is defined by

$$h(x) = \frac{1 - \exp[\psi(x - \omega)]}{1 + \exp[\psi(x - \omega)]}, \quad \psi > 0, \quad 0 < \omega < 1$$

For the spectral coefficients of functions without shape constraints, the following prior is used (The intercept is included in  $\beta$ ):

$$\theta_j | \tau, \gamma \sim N(0, \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

The priors for the spectral coefficients of shape restricted functions are:

$$\theta_0 \sim N(m_{\theta_0}, v_{\theta_0}^2), \quad \theta_j | \tau, \gamma \sim N(m_{\theta_j}, \tau^2 \exp[-j\gamma]), \quad j \geq 1$$

To complete the model specification, the following prior is assumed for  $\beta$ :

$$\beta | \sim N(m_{0,\beta}, V_{0,\beta})$$

## Value

An object of class `bsam` representing the Bayesian spectral analysis model fit. Generic functions such as `print`, `fitted` and `plot` have methods to show the results of the fit.

The MCMC samples of the parameters in the model are stored in the list `mcmc.draws`, the posterior samples of the fitted values are stored in the list `fit.draws`, and the MCMC samples for the log marginal likelihood are saved in the list `loglik.draws`. The output list also includes the following objects:

<code>post.est</code>	posterior estimates for all parameters in the model.
<code>lmarg.gd</code>	log marginal likelihood using Gelfand-Dey method.
<code>lmarg.nr</code>	log marginal likelihood using Netwon-Raftery method, which is biased.
<code>family</code>	the family object used.
<code>link</code>	the link object used.
<code>call</code>	the matched call.
<code>mcmctime</code>	running time of Markov chain from <code>system.time()</code> .

## References

- Albert, J. H. and Chib, S. (1993) Bayesian Analysis of Binary and Polychotomous Response Data. *Journal of the American Statistical Association*, **88**, 669-679.
- Holmes, C. C. and Held, L. (2006) Bayesian Auxiliary Variables Models for Binary and Multinomial Regression. *Bayesian Analysis*, **1**, 145-168.
- Lenk, P. and Choi, T. (2017) Bayesian Analysis of Shape-Restricted Functions using Gaussian Process Priors. *Statistica Sinica*, **27**, 43-69.
- Gelfand, A. E. and Dey, K. K. (1994) Bayesian model choice: asymptotics and exact calculations. *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, **56**, 501-514.
- Newton, M. A. and Raftery, A. E. (1994) Approximate Bayesian inference with the weighted likelihood bootstrap (with discussion). *Journal of the Royal Statistical Society. Series B - Statistical Methodology*, **56**, 3-48.

## See Also

[bsaq](#), [bsar](#)

## Examples

```
## Not run:
#####
# Probit Regression Model #
#####

# Simulate data
```

```

set.seed(1)

n <- 100
x <- runif(n)
y <- log(1 + 10*x) + rnorm(n, sd = 1)

# Number of cosine basis functions
nbasis <- 50

# Fit the model with default priors and mcmc parameters
fout <- gbsar(y = y, x = x, family = binomial(link = "probit"),
             nbasis = nbasis, shape = 'IncreasingConcave')

# Summary
print(fout)

# fitted values
fit=fitted(fout)

# Plot
plot(fit,'topleft',ask=TRUE)

#####
# Logistic Additive Regression Model #
#####

# Wage-Union data
data(wage.union); attach(wage.union)

race[race==1 | race==2]=0
race[race==3]=1

y <- union
w <- cbind(race,sex,south)
x <- cbind(wage,education,age)

# mcmc parameters
mcmc <- list(blow0 = 10000,
            blow = 10000,
            nskip = 10,
            smcmc = 1000,
            ndisp = 1000,
            maxmodmet = 10)

foutGBSAR <- gbsar(y = y, w = w, x = x, family = 'binomial',
                 link = 'logit', nbasis = 50, mcmc = mcmc,
                 shape = c('Free','Decreasing','Increasing'))

# fitted values
fitGBSAR <- fitted(foutGBSAR)

# Plot
plot(fitGBSAR, ask = TRUE)

## End(Not run)

```

---

intsim                      *Numerical integration using Simpson's rule*

---

**Description**

Simpson's rule is a method for numerical integration.

**Usage**

```
intsim(f, delta)
```

**Arguments**

**f**                      Function values to be integrated.  
**delta**                 Spacing size.

**Value**

intsim returns the value of the intergral.

---

London.Mortality            *Daily Moratlity in London*

---

**Description**

The London.Mortality data consists of daily death occurrences from Jan. 1st, 1993 to Dec. 31st, 2006 and corresponding weather observations including temperature and humidity in London.

**Usage**

```
data(London.Mortality)
```

**Format**

A data frame with 5113 observations on the following 7 variables.

**date** date in YYYY-MM-DD.  
**tmean** Mean temperature.  
**tmin** Minimum dry-bulb temperature.  
**tmax** Maximum dry-bulb temperature.  
**dewp** Dew point.  
**rh** Relative humidity.  
**death** the number of death occurences.

**Source**

Office for National Statistics  
British Atmospheric Data Centre  
[https://github.com/gasparrini/2015\\_gasparrini\\_Lancet\\_Rcodedata](https://github.com/gasparrini/2015_gasparrini_Lancet_Rcodedata)

## References

Armstrong BG, Chalabi Z, Fenn B, Hajat S, Kovats S, Milojevic A, Wilkinson P (2011). Association of mortality with high temperatures in a temperate climate: England and Wales. *Journal of Epidemiology & Community Health*, **65**(4), 340–345.

Gasparrini A, Armstrong B, Kovats S, Wilkinson P (2012). The effect of high temperatures on cause-specific mortality in England and Wales. *Occupational and Environmental Medicine*, **69**(1), 56–61.

Gasparrini A, Guo Y, Hashizume M, Lavigne E, Zanobetti A, Schwartz J, Tobias A, Tong S, Rocklöv J, Forsberg B, et al.(2015). Mortality risk attributable to high and low ambient temperature: a multicountry observational study. *The Lancet*, **386**(9991), 369-375.

## Examples

```
## Not run:
data(London.Mortality)

## End(Not run)
```

---

plasma

*A Data Set for Plasma Levels of Retinol and Beta-Carotene*

---

## Description

This data set contains 314 observations on 14 variables.

## Usage

```
data(plasma)
```

## Format

age Age (years).  
sex Sex (1=Male, 2=Female).  
smoke Smoking status (1=Never, 2=Former, 3=Current Smoker).  
vmi BMI values (weight/(height^2)).  
vitas Vitamin use (1=Yes, fairly often, 2=Yes, not often, 3=No).  
calories Number of calories consumed per day.  
fat Grams of fat consumed per day.  
fiber Grams of fiber consumed per day.  
alcohol Number of alcoholic drinks consumed per week.  
cholesterol Cholesterol consumed (mg per day).  
beta diet Dietary beta-carotene consumed (mcg per day).  
reedit Dietary retinol consumed (mcg per day).  
betaplasma Plasma beta-carotene (ng/ml).  
retplasma Plasma Retinol (ng/ml).

**Source**

<http://staff.pubhealth.ku.dk/~tag/Teaching/share/data/Plasma.html>

**References**

Nierenberg, D. W., Stukel, T. A., Baron, J. A., Dain, B. J., and Greenberg, E. R. (1989). Determinants of plasma levels of beta-carotene and retinol. *American Journal of Epidemiology*, **130**, 511-521.

Meyer, M. C., Hackstadt, A. J., and Hoeting, J. A. (2011). Bayesian estimation and inference for generalized partial linear models using shape-restricted splines. *Journal of Nonparametric Statistics*, **23**(4), 867-884.

**Examples**

```
data(plasma)
```

---

plot.blm

*Plot a blm object*

---

**Description**

Plots the posterior samples for Bayesian linear models

**Usage**

```
## S3 method for class 'blm'  
plot(x, ...)
```

**Arguments**

x	a blm object
...	other options to pass to the plotting functions

**Value**

Returns a plot.

**See Also**

[blq](#), [blr](#)

**Examples**

```
## See examples for blq and blr
```

---

plot.bsam	<i>Plot a bsam object</i>
-----------	---------------------------

---

**Description**

Plots the posterior samples for Bayesian spectral analysis models.

**Usage**

```
## S3 method for class 'bsam'  
plot(x, ...)
```

**Arguments**

x	a bsam object
...	other options to pass to the plotting functions

**Value**

Returns a plot.

**See Also**

[bsaq](#), [bsaqdpm](#), [bsar](#), [bsardpm](#)

**Examples**

```
## See examples for bsaq, bsaqdpm, bsar, and bsardpm
```

---

plot.bsamdpm	<i>Plot a bsamdpm object</i>
--------------	------------------------------

---

**Description**

Plots the posterior samples for Bayesian spectral analysis models with Dirichlet process mixture error.

**Usage**

```
## S3 method for class 'bsamdpm'  
plot(x, ...)
```

**Arguments**

x	a bsamdpm object
...	other options to pass to the plotting functions

**Value**

Returns a plot.

**See Also**

[bsaqdpm](#), [bsardpm](#)

**Examples**

```
## See examples for bsaqdpn and bsardpm
```

---

plot.fitted.bsad      *Plot a fitted.bsad object*

---

**Description**

Plots the predictive density for Bayesian density estimation model using logistic Gaussian process

**Usage**

```
## S3 method for class 'fitted.bsad'
plot(x, ...)
```

**Arguments**

x                    a fitted.bsad object  
 ...                  other options to pass to the plotting functions

**Value**

Returns a plot.

**See Also**

[bsad](#), [fitted.bsad](#)

**Examples**

```
## See example for bsad
```

---

plot.fitted.bsam      *Plot a fitted.bsam object*

---

**Description**

Plots the data and the fit for Bayesian spectral analysis models.

**Usage**

```
## S3 method for class 'fitted.bsam'
plot(x, ask, ...)
```



**Arguments**

x                    a fitted.bsam object  
ask                  see. [par](#)  
...                  other options to pass to the plotting functions

**Value**

Returns a plot.

**See Also**

[bsaq](#), [bsaqdpm](#), [bsar](#), [bsardpm](#), [fitted.bsam](#)

**Examples**

```
## See examples for bsaq, bsaqdpm, bsar, and bsardpm
```

---

`plot.fitted.bsamdpm`    *Plot a fitted.bsamdpm object*

---

**Description**

Plots the data and the fit for Bayesian spectral analysis models with Dirichlet process mixture error.

**Usage**

```
## S3 method for class 'fitted.bsamdpm'  
plot(x, ask, ...)
```

**Arguments**

x                    a fitted.bsamdpm object  
ask                  see. [par](#)  
...                  other options to pass to the plotting functions

**Value**

Returns a plot.

**See Also**

[bsaqdpm](#), [bsardpm](#), [fitted.bsamdpm](#)

**Examples**

```
## See examples for bsaqdpm and bsardpm
```

rald

*The asymmetric Laplace distribution***Description**

Density for and random values from a three-parameter asymmetric Laplace distribution.

**Usage**

```
rald(n, location=0, scale=1, p=0.5)
```

**Arguments**

n	Number of random values to be generated.
location	Location parameter.
scale	Scale parameter.
p	Skewness parameter.

**Details**

This generic function generates a random variable from an asymmetric Laplace distribution (ALD). The ALD has the following probability density function:

$$ALD_p(x; \mu, \sigma) = \frac{p(1-p)}{\sigma} \exp\left(-\frac{(x-\mu)[p - I(x \leq \mu)]}{\sigma}\right),$$

where  $0 < p < 1$  is the skew parameter,  $\sigma > 0$  is the scale parameter,  $-\infty < \mu < \infty$  is the location parameter, and  $I(\cdot)$  is the indication function. The range of  $x$  is  $(-\infty, \infty)$ .

**Value**

rald gives out a vector of random numbers generated by the asymmetric Laplace distribution.

**References**

- Koenker, R. and Machado, J. (1999). Goodness of fit and related inference processes for quantile regression. *Journal of the American Statistical Association*, **94**(3), 1296-1309.
- Yu, K. and Zhang, J. (2005). A Three-parameter asymmetric Laplace distribution and its extension. *Communications in Statistics - Theory and Methods*, **34**, 1867-1879.

---

traffic	<i>Monthly traffic accidents data</i>
---------	---------------------------------------

---

**Description**

This data set contains 108 observations on 6 variables.

**Usage**

```
data(traffic)
```

**Format**

ln\_number logarithm of the number of monthly automobile accidents in the state of Michigan.

month months from January 1st, 1979 to December 31st, 1987.

ln\_unemp logarithm of unemployment rate

spring indicator for spring season.

summer indicator for summer season.

fall indicator for fall season.

**References**

Lenk (1999) Bayesian inference for semiparametric regression using a Fourier representation. *Journal of the Royal Statistical Society: Series B*, **61**(4), 863-879.

**Examples**

```
## Not run:  
data(traffic)  
pairs(traffic)  
  
## End(Not run)
```

---

wage.union	<i>Wage-Union data</i>
------------	------------------------

---

**Description**

This data set contains 534 observations on 11 variables.

**Usage**

```
data(wage.union)
```

**Format**

education number of years of education.

south indicator of living in southern region of U.S.A.

sex gender indicator: 0=male,1=female.

experience number of years of work experience.

union indicator of trade union membership: 0=non-member, 1=member.

wage wages in dollars per hour.

age age in years.

race 1=black, 2=Hispanic, 3=white.

occupation 1=management, 2=sales, 3=clerical, 4=service, 5=professional, 6=other.

sector 0=other, 1=manufacturing, 2=construction.

married indicator of being married: 0=unmarried, 1=married.

**References**

Berndt, E.R. (1991) *The Practice of Econometrics*. New York: Addison-Wesley.

Ruppert, D., Wand, M.P. and Carroll, R.J. (2003) *Semiparametric Regression*. Cambridge University Press.

**Examples**

```
## Not run:  
data(wage.union)  
pairs(wage.union)  
  
## End(Not run)
```

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