Example on Bayesian Spectral Analysis Quantile Regression

Statlab2

2017-08-04

Preliminaries

To use `bsamGP`, you need to first install the package. Use RGui menu "Package/Install package(s) from local zip files…" or R command "install.packages".

You then load the `bsamGP` package using the `library` or `require` function:

```
library(bsamGP)
```

This needs to be done every time you start R.

To get help on the functions in R (and in `bsamGP`), use `help()` or `?`. For example, to view the help file for the `bsaq` function, type one of the following:

```
help(bsaq) # ?bsaq
```

Bayesian Spectral Analysis Quantile Regression (BSAQ)

Let's now proceed to the 'S' shape-restricted function estimating via the following BSAQ model.

$$Y_i = \boldsymbol{w}_i^\top \boldsymbol{\beta} + f(x_i) + \epsilon_i, \;\; \epsilon_i \sim ALD(p; 0, \sigma), \tag{1}$$

where p is a quantile of interest; $\boldsymbol{w}_i$ and $\boldsymbol{\beta}$ are $q+1$–dimensional vectors of covariates and coefficients; f is an unknown function of the scalar x_i estimated by

$$\begin{aligned} f(x) &= \delta\zeta \int_0^x \int_0^s Z^2(t)h(t)dt\,ds - \delta\xi(x-0.5) + \alpha(x-0.5) \\ \xi &= \min\left[0, \min_{x\in[0,1]} \zeta \int_0^x Z^2(s)h(s)ds\right] \\ \varphi^b_{j,k}(x) &= \int_0^x \int_0^s \varphi_j(t)\varphi_k(t)dt\,ds - \int_0^1 \int_0^x \int_0^s \varphi_j(t)\varphi_k(t)dt\,ds\,dx. \end{aligned}$$

The model (1) can be represented by a location-scale mixture of exponential and normal distributions as follows.

$$\begin{aligned} &Y_i = \boldsymbol{w}_i^T \boldsymbol{\beta} + f(x_i) + \eta_1 \nu_i + \epsilon_i, \;\; i = 1, \ldots, n, \\ &\epsilon_i \sim N(0, \sigma^2\eta_2^2 z_i), \;\; \nu_i = \sigma z_i, \;\; z_i \sim Exp(1), \;\; i = 1, \ldots, n, \end{aligned}$$

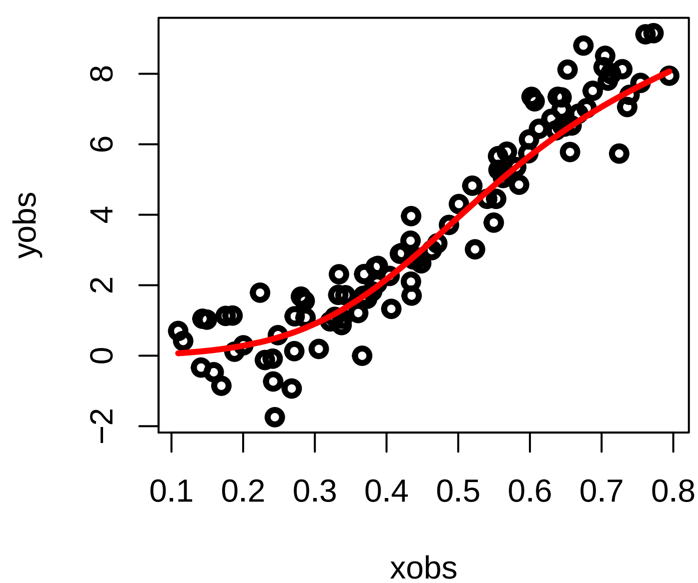
where $\boldsymbol{w}_i$ and $\boldsymbol{\beta}$ are $(p+1)$-dimensional vectors of covariates and coefficients, $\eta_1 = (1-2p)/\{p(1-p)\}$ and $\eta_2^2 = 2/\{p(1-p)\}$.

Data generation

We will consider estimating the following 'S' shape-restricted function where $x \in [0,1]$.

$$f(x) = 5\exp(-10(x-1)^4) + 5x^2$$

```
set.seed(1)
ftn = function(x) 5 * exp(-10 * (x-1)^4)+5 * x^2
n = 100
xmin = 0.1; xmax = 0.8;
xobs = xmin+(xmax-xmin) * runif(n)
xobs = sort(xobs)
yobs = ftn(xobs) + rald(n,p=0.5)
plot(xobs,yobs,lwd=3)
lines(xobs,ftn(xobs),lwd=3,col=2)
```



Model Fitting

To fit BSAQ model, we first set up the MCMC parameters, the number of basis and prior information for spectral coefficients.

```
# MCMC parameters
nblow0 = 1000; # Initialization period for adpative metropolis
nblow = 10000; # Number of MCMC in transition period
smcmc = 2000; # Number of MCMC for analysis
nskip = 10; # Number of MCMC to skip after nblow
ndisp = 1000; # number of saved draws to be displayed on screen
maxmodmet = 5; # Maximum number of times to modify metropolis
```

```r
# Prior information
iflagprior = 0; # 1 = Lasso Smoother, 0 = T Smoother
nbasis = 30; # number of cosine basis functions @

# quantile of interest
p=0.5 # median regression
```

To generate a posterior sample for the Bayesian semiparametric quantile regression model, use the function bsaq.

```r
# Fit the model
fout = bsaq(y=yobs, x=xobs, w=NULL, shape='IncreasingS', p=p,
           xmin=xmin, xmax=xmax, nbasis=nbasis,
           mcmc=list(maxmodmet=maxmodmet,nblow0=nblow0,nblow=nblow,smcmc=smcmc,nskip=nskip,ndisp=ndisp)
           prior=list(iflagprior=iflagprior))
```

```
## Initializing MCMC parameters ...
## function[1]: pmet = 0.2760 < 0.3. Reduce metm and redo MCMC loop
## function[1]: pmet = 0.6620 > 0.6. Increase metm and redo MCMC loop
## Burnin ...
## function[1]: pmet = 0.3081
## Main iterations ...
## MCMC draws 1000 of 2000 (CPU time: 21.266 s)
## MCMC draws 2000 of 2000 (CPU time: 29.406 s)
## function[1]: pmet = 0.2601
```

The output returns bsam class object. The print function on bsam object summarizes the fit.

```r
print(fout)
```

```
##
## Call:
## bsaq.default(y = yobs, w = NULL, x = xobs, xmin = xmin, xmax = xmax,
##     p = p, nbasis = nbasis, mcmc = list(maxmodmet = maxmodmet,
##         nblow0 = nblow0, nblow = nblow, smcmc = smcmc, nskip = nskip,
##         ndisp = ndisp), prior = list(iflagprior = iflagprior),
##     shape = "IncreasingS")
##
## Bayesian Spectral Analysis Quantile Regression (BSAQ)
##
## Model:
## Y = w'beta + sum_k f_k(x_k) + ALD(p;0,sigma^2)
## xmin < x < xmax
## Normalize f: int_a^b f(x)dx = 0
## beta has the Intercept
##
## Define
## Z(x) = sum_j=0^J theta_j*phi_j(x)
## g^a(x) = int_0^x |Z(s)|^2 dx
##        = theta' Phi^a(x)*theta
## where Phi^a(x) is matrix with
## phi^a_{j,k}(x) = int_0^x phi_j(s) phi_k(s) ds in (j,k)
## g^b(x)   = int_0^x g^a(s) ds
##          = theta' Phi^b(x)*theta
## where Phi^b(x) is a matrix with
```

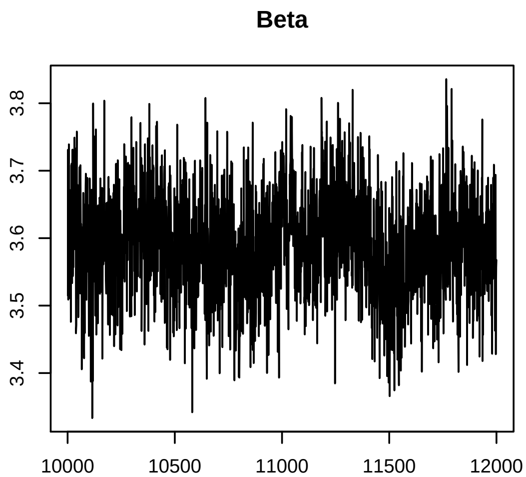
```
## phi^b_{j,k}(x) = int_0^x phi^a_{j,k}(s) ds
##
## ************************************************************************************
##
## Number of nonparametric components =  1
##
## Model for f
## Removed theta_0^2 from f but kept theta_0*theta_j
## Increasing, S shaped
## f(x) = int_a^x int_a^s Z(t)^2 h(t)dt ds +  alpha*(x-xmid) - xi*(x-xmin)
## h(t) = {1-exp[psi*(x-omega)]}/{1+exp[psi*(x-omega)]}
## psi  > 0, a < omega < b, alpha > 0
## xi = min(0,f'(x)) to make sure that f' > 0
## Model for f includes linear term alpha
##
## ************************************************************************************
##
## Cosine basis
## xrange   = xmax-xmin
## phi_0(x) = 1/sqrt(xmax-xmin) for xmin < x < xmax
## phi_k(x) = sqrt(2/(xmax-xmin))*cos(pi*k*(x-xmin)/(xmax-xmin))
##
## Scale invariant priors:
## beta|sigma       ~ N(b0,sigma2*B0)
## alpha|sigma      ~ N(m0,sigma2*v0)I(delta*alpha>0)
## theta_0|sigma    ~ N(0,sigma*v0)I(theta_0 > 0)
## theta_k|sigma    ~ N(0,sigma*tau^2*exp(-gamma*k)
##
## Smoothing parameters tau and gamma
## Choice of two priors for tau2
## 1.) T Smoother: tau2 ~ IG(r0/2,s0/2)
## gamma ~ Exp(w0)
##
## Note: posterior of tau and gamma have banana contours
## zeta = ln(tau2) - kbar*gamma is less dependent with gamma or log(gamma)
##
## S models uses squish function (reparameterization of hyperbolic tangent)
## that depens on slope psi and location omega (inflection point for S).
## psi  ~ N(m0,v0)I(psi > 0)  Truncated Normal
## omega    ~ N(m0,v0)I(xmin < omega < xmax) and m0 = (xmin+xmax)/2
##
## ************************************************************************************
##
## Quantile of interest                =  0.5
## Number of Cosine basis functions    =  30
## Number of observations              =  100
## Number of covariates (no intercept)  =  0
##
## MCMC transition draws               =  10000
## MCMC draws saved for estimation     =  2000
## Save every nskip draws              =  10
## MCMC draws total                    =  30000
##
## Function =  1
```

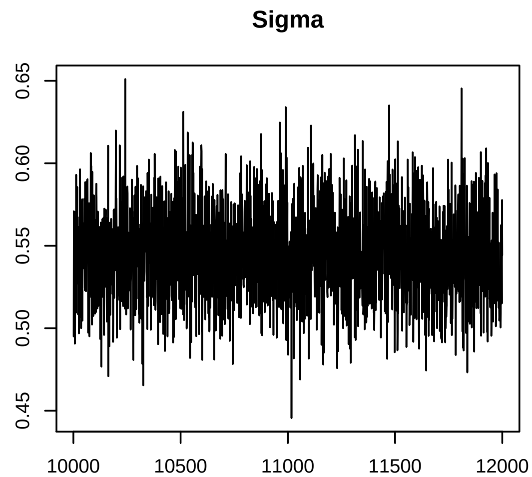
```
## Proportion of Theta Accepted after burn-in    =  0.26015
##
## R-Square                        =  0.9357
##
## Log Integrated Likelihood
## LIL Gelfand & Dey               =  -149.2621
## LIL Newton & Raftery (biased)  =  -118.5313
##
## beta
## const is Y intercept
##          PostM    PostStd PostM/STD
## const 3.590208 0.07820325  45.90868
##
## sigma
## PostM sigma  =  0.5445777
## PostS sigma  =  0.02774913
##
## ****************************************************************************************
##
## Function = 1
## Increasing S (convex to concave) function
##
## Linear term alpha in x for constrainted f
## Posterior mean   of alpha =  1.268502
## Posterior stddev of alpha =  0.899959
##
## psi is slople of squish function
## Posterior mean   of psi =  99.99063
## Posterior stddev of psi =  0.2547264
##
## omega is inflection point of squish function
## Posterior mean  omega =  0.5518184
## Posterior stdev omega =  0.02345653
##
## theta_k ~ N(0,sigma*tau2*exp(-gamma*k))
##
## Tau
## PostM   PostS
## 1.094411 0.4684179
##
## Gamma
##      PostM      PostS
## 0.65154636 0.05370321
##
## Zeta = ln(tau2) - mean(k)*gamma
##      PostM      PostS
## -9.7525902  0.3811029
##
## Cosine Basis weights theta
##     PostMean PostSTD    Ratio
## T0   6.09456 0.38012 16.03306
## T1  -0.57424 0.45535 -1.26109
## T2   0.04237 0.32366  0.13093
## T3  -0.20269 0.34984 -0.57939
```

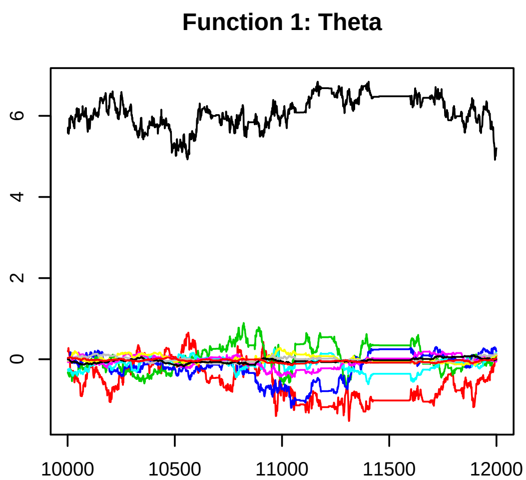
```
## T4  -0.13233 0.16296 -0.81199
## T5  -0.04989 0.14613 -0.34141
## T6   0.00268 0.08663  0.03092
## T7  -0.01732 0.06186 -0.28004
## T8  -0.03979 0.05442 -0.73119
## T9  -0.04268 0.03767 -1.13286
## T10 -0.01749 0.02610 -0.66991
## T11  0.00524 0.02247  0.23331
## T12  0.00493 0.01341  0.36731
## T13  0.00007 0.00962  0.00730
## T14  0.00526 0.00529  0.99423
## T15 -0.00376 0.00691 -0.54441
## T16  0.00062 0.00391  0.15765
## T17 -0.00097 0.00268 -0.36325
## T18  0.00011 0.00242  0.04580
## T19  0.00053 0.00121  0.43431
## T20  0.00002 0.00082  0.02153
## T21 -0.00008 0.00079 -0.10612
## T22  0.00003 0.00056  0.06048
## T23  0.00010 0.00038  0.26192
## T24 -0.00015 0.00027 -0.56616
## T25 -0.00006 0.00026 -0.24446
## T26  0.00003 0.00024  0.14149
## T27 -0.00006 0.00009 -0.69569
## T28 -0.00002 0.00009 -0.23941
## T29  0.00001 0.00006  0.15423
## T30  0.00002 0.00004  0.44088
## ***************************************************************************************
```

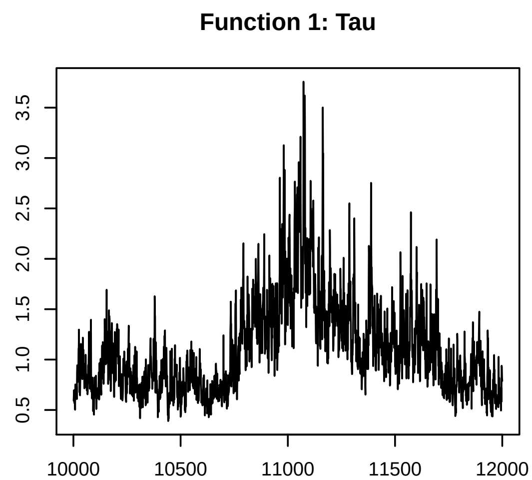
The `plot` function on `bsam` object returns traceplots of each parameters for diagnostics.

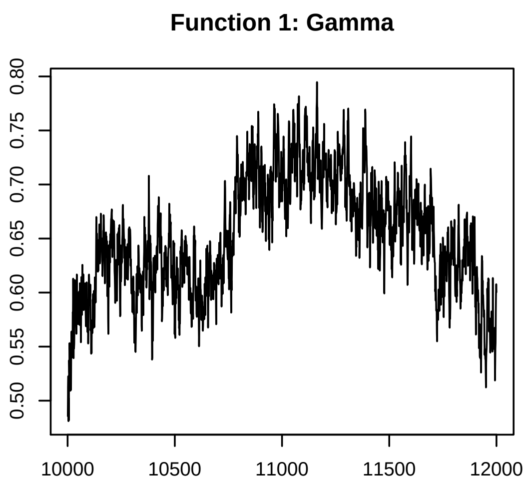
```
plot(fout)
```

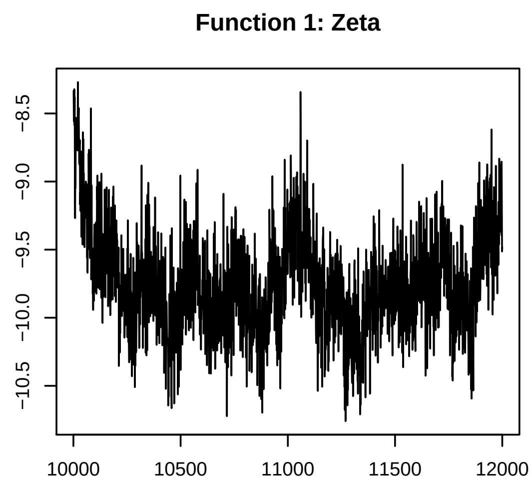

Beta
3.4
3.5
3.6
3.7
3.8
10000
10500
11000
11500
12000


Sigma
0.45
0.50
0.55
0.60
0.65
10000
10500
11000
11500
12000


Function 1: Theta
0
2
4
6
10000
10500
11000
11500
12000

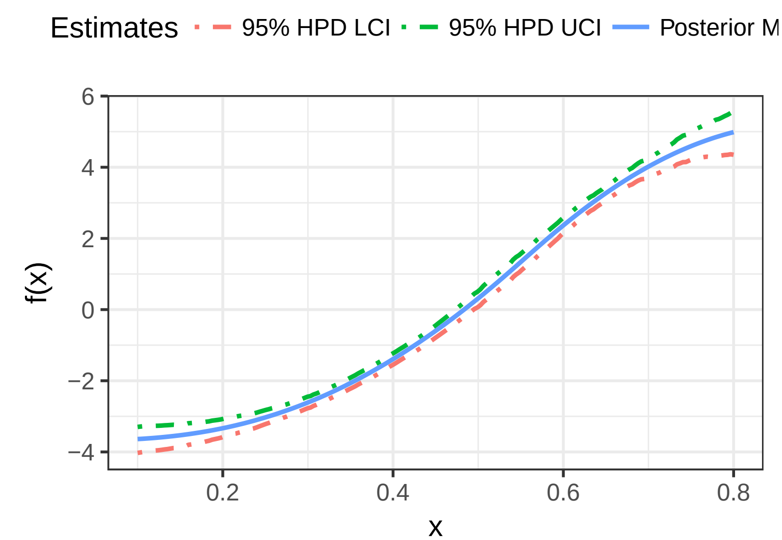

Function 1: Tau
0.5
1.0
1.5
2.0
2.5
3.0
3.5
10000
10500
11000
11500
12000

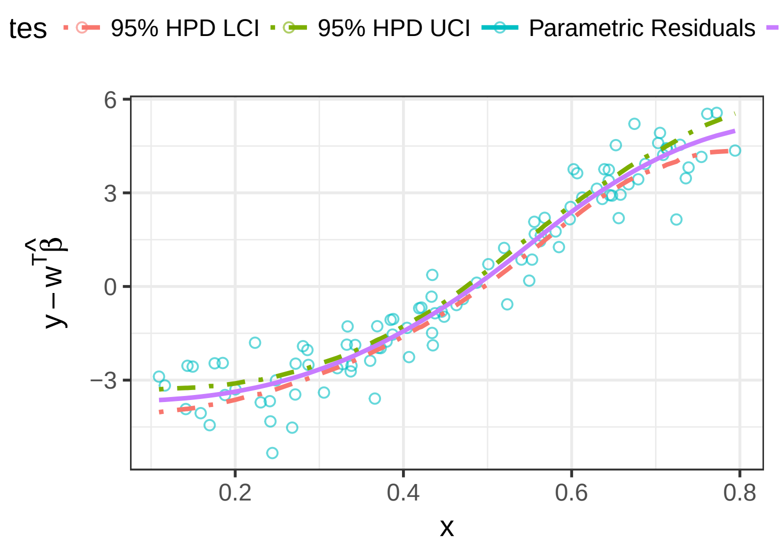

Function 1: Gamma
0.50
0.55
0.60
0.65
0.70
0.75
0.80
10000
10500
11000
11500
12000


Function 1: Zeta
−10.5
−10.0
−9.5
−9.0
−8.5
10000
10500
11000
11500
12000

We may visualize fitted median curve and 95% highest posterior density (HPD) interval with `plot` function on fitted object from `fitted` method.

```r
fit = fitted(fout)
plot(fit)
```





For more detailed examples and real data applications, see Jo, S., Choi, T., Park, B., & Lenk, P. (2017) "bsamGP: An R package for Bayesian Spectral Analysis Models using Gaussian Process Priors", *Preprint* .